INDIAN STATISTICAL INSTITUTE, CHENNAI CENTRE

Analysis-II

Quiz II **Instructor: S. Ponnusamy**

Date: 01-04-2016 **Time: 10.00-12.00 am** **Total Marks: 20**

Instructions:

- Write your roll number and your name in the answer book
- Use of calculator, mobile phone and mathematical table is not allowed
- Justify your answers by clearly stating the appropriate results/theorems that you use whenever required

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1. Write down the precise definition of continuity and differentiability of a vector-valued function defined on $\mathbb{R}^n$, $n > 1$. **(2 marks)**

2. Give an example of a real-valued function defined on $\mathbb{R}^2$ which is discontinuous at a point but have directional derivative in every unit direction. **(3 marks)**

3. Suppose that $f : \mathbb{R}^n \to \mathbb{R}$ is differentiable. Show that f has directional derivative in every unit direction e at each $a \in \mathbb{R}^n$. Show also that
$$D_e f(a) = Df(a)e,$$
where $D_e f(a)$ and $Df(a)$ denote respectively the directional derivative and total derivative of f evaluated at a. **(4 marks)**

4. Consider the function
$$f(x, y) = (2x^e + ey^2 + e|xy|, \pi x^2 + 2y^\pi + \pi|xy|)$$
Describe the set Ω of all points in $\mathbb{R}^2$ on which f is differentiable and write down the total derivative of f at points in Ω. **(3 marks)**

5. Consider
$$f(x, y) = \begin{cases} \dfrac{xy(x+y)}{y-x} & \text{if } x \neq y \\ 0 & \text{if } x = y. \end{cases}$$
Is f continuous at$(0, 0)$? Do $f_x(0, 0)$ and $f_y(0, 0)$ exist? Do $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ coincide? Answer the same at points of the form $(x, x), x \in \mathbb{R}$. **(5 marks)**

6. State and prove the higher dimensional version of mean value theorem for vector-valued function. **(3 marks)**

Bonus Mark Questions

7. Is it possible to define the function at origin so that $e^{-\frac{1}{\|x\|^2}}$ is continuous. **(2 marks)**

8. Does there exist a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that f_{yx} exists on $\mathbb{R}^2$ but f_x exists nowhere. **(3 marks)**